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Assisted Cloning and Orthogonal Complementing of an Arbitrary Unknown Two-qubit Entangled State via Positive Operator-valued Measure

Niu Xiao-Fei

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Abstract By using positive operator-valued measure (POVM) instead of the more commonly used collective unitary operation in the quantum teleportation process, Zhan's scheme (Zhan in Phys. Lett. A 336:317, 2005) is generalized to treat an *arbitrary* (*not a special class of*) unknown two-qubit entangled state. Generally speaking, conditioned on Victor's classical message, Alice can create either a copy or an orthogonal-complement copy of the original state. However, it is shown that Alice's perfect clone can be realized with higher probability or even in a deterministic manner via an appropriate unitary operation provided that the state to be cloned is chosen from six special ensembles.

Keywords Quantum cloning · Arbitrary two-qubit entangled state · Two-qubit projective measurement · Positive operator-valued measure · Unitary operation

The principles of quantum mechanics supplied many interesting application in the field of information in the last decade, such as quantum computer [1, 2], quantum cryptography [2–5], quantum teleportation [6], quantum secret sharing (QSS) [7–14], and so on. One of the greatest differences between classical and quantum information is that while classical information can be copied perfectly, quantum cannot. In particular, one can not create a duplicate of an arbitrary qubit without destroying the original one [15]. This follows from the no-cloning theorem of Wootters and Zurek [16] (see also [17, 18]). However, quantum cloning or approximate cloning is necessary in some quantum information processes [19]. Hence, in literatures various approximate cloning machines have been proposed [20–34], which operate either in a deterministic or probabilistic way. Very recently Zhan [35] has proposed two quantum cloning protocols. In his first protocol, two maximally entangled two-qubit states (i.e., Bell states) are taken as quantum channel, and a perfect clone of a special class of unknown two-qubit entangled state (i.e., $\alpha|00\rangle + \beta|11\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$) or implementation of its complement state is realized with assistance offered by the unknown

N. Xiao-Fei (✉)
Department of Electronics and Electrical Engineering, Suzhou University,
Suzhou, Anhui 234000, China
e-mail: niuxf@yahoo.com.cn

state preparer. In his second protocol, non-maximally entangled two-qubit states are used as quantum channel, a probabilistic clone of the special class of unknown two-qubit entangled state (i.e., $\alpha|00\rangle + \beta|11\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$) or implementation of its complement state can be realized with assistance offered by the unknown state preparer. However, it is different from the general clone protocols, such as universal and phase-covariant quantum clone [33, 34]. Actually, It consists of two processes, quantum teleportation and remote state preparation. In quantum teleportation, the state to be cloned is transmitted via entanglement swapping, while the state to be cloned is reconstructed with the state preparer's assistance in remote state preparation. In this letter, by introducing POVM [36–39] in quantum teleportaion process, I will generalize the two protocols to treat an *arbitrary* (*not special class of*) unknown two-qubit entangled state (i.e., $\alpha|00\rangle + \beta|11\rangle + \gamma|01\rangle + \delta|10\rangle$, where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$).

Along the line of Zhan's protocols [35], I present in detail my generalized protocols as follows. Let us first consider the case that maximally-entangled two-qubit pairs are taken as quantum channel. Suppose there are two participants, Victor and Alice. Victor is the so-called state preparer. He prepares a two-qubit state

$$|u\rangle = \alpha|00\rangle_{12} + \beta|11\rangle_{12} + \gamma|01\rangle_{12} + \delta|10\rangle_{12}, \tag{1}$$

where 1 and 2 label the two qubits, $\alpha, \beta, \gamma, \delta$ are arbitrary complex numbers and they satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. This state is completely unknown and arbitrary for Alice. Alice has safely received it from Victor and will use it as her input state. Alice wishes to create either a copy or a complement copy of this unknown state with the help of Victor. Besides the qubit pair $(1, 2)$ from Victor, Alice has another two qubit pairs $(3, 4)$ and $(5, 6)$. The two pairs $(3, 4)$ and $(5, 6)$ are in same Bell states. Without loss of generality, suppose they are

$$|\psi^-\rangle_{34} = (|01\rangle_{34} - |10\rangle_{34})/\sqrt{2}, \qquad |\psi^-\rangle_{56} = (|01\rangle_{56} - |10\rangle_{56})/\sqrt{2}. \tag{2}$$

Incidentally, another three Bell states are defined as

$$|\psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}, \qquad |\phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}. \tag{3}$$

Hence the joint state of the six qubits in Alice's site is

$$\begin{aligned} |w\rangle_{123456} &= |u\rangle_{12}|\psi^-\rangle_{34}|\psi^-\rangle_{56} \\ &= (\alpha|00\rangle_{12} + \beta|11\rangle_{12} + \gamma|01\rangle_{12} + \delta|10\rangle_{12})(|01\rangle_{34} \\ &\quad - |10\rangle_{34})(|01\rangle_{56} - |10\rangle_{56})/2. \end{aligned} \tag{4}$$

It can be rewritten as

$$\begin{aligned} |w\rangle_{123456} = \frac{1}{4}(&|\psi^-\rangle_{25}|\psi^-\rangle_{13}I|u\rangle_{46} - |\psi^-\rangle_{25}|\psi^+\rangle_{13}\sigma_4^z|u\rangle_{46} \\ &- |\psi^-\rangle_{25}|\phi^-\rangle_{13}\sigma_4^x|u\rangle_{46} - |\psi^-\rangle_{25}|\phi^+\rangle_{13}\sigma_4^z\sigma_4^x|u\rangle_{46}) \\ &+ \frac{1}{4}(-|\psi^+\rangle_{25}|\psi^-\rangle_{13}\sigma_6^z|u\rangle_{46} + |\psi^+\rangle_{25}|\psi^+\rangle_{13}\sigma_6^z\sigma_4^z|u\rangle_{46} \\ &+ |\psi^+\rangle_{25}|\phi^-\rangle_{13}\sigma_6^z\sigma_4^x|u\rangle_{46} + |\psi^+\rangle_{25}|\phi^+\rangle_{13}\sigma_6^z\sigma_4^z\sigma_4^x|u\rangle_{46}) \\ &- \frac{1}{4}(|\phi^-\rangle_{25}|\psi^-\rangle_{13}\sigma_6^x\sigma_4^x|u\rangle_{46} - |\phi^-\rangle_{25}|\psi^+\rangle_{13}\sigma_6^x\sigma_4^x\sigma_4^z|u\rangle_{46} \end{aligned}$$

$$
\begin{aligned}
&- |\phi^-\rangle_{25}|\phi^-\rangle_{13}\sigma_6^x|u\rangle_{46} + |\phi^-\rangle_{25}|\phi^+\rangle_{13}\sigma_6^x\sigma_4^z|u\rangle_{46}) \\
&- \frac{1}{4}(-|\phi^+\rangle_{25}|\psi^-\rangle_{13}\sigma_6^x\sigma_6^z\sigma_4^x|u\rangle_{46} + |\phi^+\rangle_{25}|\psi^+\rangle_{13}\sigma_6^x\sigma_6^z\sigma_4^x\sigma_4^z|u\rangle_{46} \\
&+ |\phi^+\rangle_{25}|\phi^-\rangle_{13}\sigma_6^x\sigma_6^z|u\rangle_{46} - |\phi^+\rangle_{25}|\phi^+\rangle_{13}\sigma_6^z\sigma_6^x\sigma_4^z|u\rangle_{46}),
\end{aligned}
\tag{5}
$$

where I is an identity operator and σ's are Pauli operators.

Assume that Alice performs projective Bell-state measurements on the qubit pairs $(2,5)$ and $(1,3)$ respectively, and if Alice's measurement outcome is $|\psi^+\rangle_{25}|\psi^-\rangle_{13}$ (the probability of such result is only 1/16), then the resultant six-qubit state can be written as

$$
|\psi^+\rangle_{25}\langle\psi^+|\psi^-\rangle_{13}\langle\psi^-|w\rangle_{123456} = -\frac{1}{4}|\psi^+\rangle_{25}|\psi^-\rangle_{13}\sigma_6^z|u\rangle_{46}. \tag{6}
$$

According to her measurement outcome, Alice needs to perform a unitary operation σ^z on her qubit 6 to get the original unknown state $|u\rangle$ in her qubits 4 and 6. In fact, this is in essence the quantum teleportation process of an arbitrary unknown two-qubit entangled state.

To create either a copy or an orthogonal-complement copy of the arbitrary unknown state $|u\rangle$, Alice needs the help of Victor. According to the projection postulate of quantum mechanics, if Alice applies projector $|\psi^+\rangle_{25}\langle\psi^+|\psi^-\rangle_{13}\langle\psi^-|$ onto the initial joint state $|w\rangle$, the state of qubits 1, 2, 3 and 5 will collapse in the state $|\psi^+\rangle_{25}|\psi^-\rangle_{13}$. Alice sends the qubits 1 and 2 to Victor and keeps the qubits 3 and 5 in her possession. Since Victor as the preparer knows the parameters α, β, γ and δ of the original state $|u\rangle$ completely, he carries out a two-qubit projective measurement on the qubit 1 and 2 in a set of mutually orthogonal basis vectors $\{|\xi_1\rangle, |\xi_2\rangle, |\xi_3\rangle, |\xi_4\rangle\}$, which are given

$$
|\xi_1\rangle = \alpha|00\rangle + \beta|11\rangle + \gamma|01\rangle + \delta|10\rangle \equiv |u\rangle, \tag{7}
$$

$$
|\xi_2\rangle = \eta\alpha|00\rangle + \eta\beta|11\rangle - \eta^{-1}\gamma|01\rangle - \eta^{-1}\delta|10\rangle, \tag{8}
$$

$$
|\xi_3\rangle = \beta^*|00\rangle - \alpha^*|11\rangle + \delta^*|01\rangle - \gamma^*|10\rangle, \tag{9}
$$

$$
|\xi_4\rangle = \eta\beta^*|00\rangle - \eta\alpha^*|11\rangle - \eta^{-1}\delta^*|01\rangle + \eta^{-1}\gamma^*|10\rangle, \tag{10}
$$

where $\eta = \sqrt{\frac{1-p}{1+p}}$ and $p = |\alpha|^2 + |\beta|^2 - |\gamma|^2 - |\delta|^2$. This four non-maximally entangled basis states $\{|\xi_1\rangle, |\xi_2\rangle, |\xi_3\rangle, |\xi_4\rangle\}$ are related to the computation basis vector $\{|00\rangle, |11\rangle, |01\rangle, |10\rangle\}$ and form a complete orthogonal basis set in a four-dimensional Hilbert space, i.e., $\langle\xi_i|\xi_j\rangle = \delta_{ij}$. Thus, the entangled state $|\psi^+\rangle_{25}|\psi^-\rangle_{13}$ in the basis $\{|\xi_1\rangle, |\xi_2\rangle, |\xi_3\rangle, |\xi_4\rangle\}$ can be written as

$$
|\psi^+\rangle_{25}|\psi^-\rangle_{13} = -\frac{1}{2}[|\xi_1\rangle_{12}|\xi_3\rangle_{35} - |\xi_3\rangle_{12}|\xi_1\rangle_{35} + |\xi_2\rangle_{12}|\xi_4\rangle_{35} - |\xi_4\rangle_{12}|\xi_2\rangle_{35}]. \tag{11}
$$

If the outcome of Victor's measurement on the two qubits 1 and 2 is $|\xi_3\rangle_{12}$, (6) can be written as

$$
|\xi_3\rangle_{12}\langle\xi_3|\psi^+\rangle_{25}\langle\psi^+|\psi^-\rangle_{13}\langle\psi^-|w\rangle_{123456} = -\frac{1}{8}|\xi_3\rangle_{12}|\xi_1\rangle_{35}\sigma_6^z|u\rangle_{46} = -\frac{1}{8}|\xi_3\rangle_{12}|u\rangle_{35}\sigma_6^z|u\rangle_{46}. \tag{12}
$$

Victor sends the measurement outcome to Alice through classical channel with two classical bits, then Alice knows that the state of her qubits 3 and 5 has already been in the state $|u\rangle_{35}$, which is just a copy of the original arbitrary unknown state $|u\rangle_{12}$. In this case, Alice has realized a perfect clone with assistance of Victor. If Alice's measurement outcome is $|\xi_1\rangle_{12}$, then (6) can be written as

$$|\xi_1\rangle_{12}\langle\xi_1|\psi^+\rangle_{25}\langle\psi^+|\psi^-\rangle_{13}\langle\psi^-|w\rangle_{123456} = -\frac{1}{8}|\xi_1\rangle_{12}|\xi_3\rangle_{35}\sigma_6^z|u\rangle_{46}. \tag{13}$$

After obtaining two cubits from Victor, Alice knows in her qubits 3 and 5 she has got a complement copy $|\xi_3\rangle_{35}$ of the original arbitrary unknown state $|u\rangle_{12}$. Similarly, if Alice's measurement outcomes is $|\xi_2\rangle_{12}$ or $|\xi_4\rangle_{12}$, then (6) can be written as, respectively

$$|\xi_2\rangle_{12}\langle\xi_2|\psi^+\rangle_{25}\langle\psi^+|\psi^-\rangle_{13}\langle\psi^-|w\rangle_{123456} = -\frac{1}{8}|\xi_2\rangle_{12}|\xi_4\rangle_{35}\sigma_6^z|u\rangle_{46}, \tag{14}$$

$$|\xi_4\rangle_{12}\langle\xi_4|\psi^+\rangle_{25}\langle\psi^+|\psi^-\rangle_{13}\langle\psi^-|w\rangle_{123456} = -\frac{1}{8}|\xi_4\rangle_{12}|\xi_2\rangle_{35}\sigma_6^z|u\rangle_{46}. \tag{15}$$

From the two equations one can easily find that, after Victor's two-qubit projective measurement mentioned above, Alice gets either the complement copy $|\xi_4\rangle_{35}$ or the complement copy $|\xi_2\rangle_{35}$ of the original arbitrary unknown state $|u\rangle_{12}$.

As we saw just, in the later three cases the collapsed state can not be converted into the wanted state. However, it should be noted that the coefficients α, β, γ and δ are assumed to be complex in the very beginning. Therefore one may ask, if α, β, γ and δ are some special values, can the collapsed states, in the later three cases, be converted into the wanted state via appropriate local unitary operations? My investigation indicates the answer is positive. The special coefficients are found out in this paper, and classified into six types. This can be seen in the following.

Case I: α, β, γ and δ are real

In this case, If Victor gets $|\xi_1\rangle_{12}$, then according to (11), after getting Victor's classical message, Alice knows the collapsed state of qubits 3 and 5 is $\beta|00\rangle_{35} - \alpha|11\rangle_{35} + \delta|01\rangle_{35} - \gamma|10\rangle_{35}$ (except for a overall trivial factor). Alice can convert it into the state she wants to copy by performing the local unitary operation $U_1 = (|1\rangle_3\langle 0| - |0\rangle_3\langle 1|)(|1\rangle_5\langle 0| + |0\rangle_5\langle 1|)$. If Victor gets $|\xi_2\rangle_{12}$ or $|\xi_4\rangle_{12}$, in both cases, Alice knows in her qubits 3 and 5 she has got a orthogonal-complement copy $|\xi_4\rangle_{35}$ and $|\xi_2\rangle_{35}$ of the original arbitrary unknown state $|u\rangle_{12}$, respectively.

Case II: α, β, γ and δ satisfy $\eta = 1$

In this case, if Victor's measurement result is $|\xi_4\rangle_{12}$, then the collapsed state of qubits 3 and 5, according to the equation 11, is $\alpha|00\rangle_{35} + \beta|11\rangle_{35} - \gamma|01\rangle_{35} - \delta|10\rangle_{35}$ (except for a overall trivial factor). With Victor's help, Alice can also copy the original state in her qubits 3 and 5 by performing the local unitary operation $U_2 = (|0\rangle_3\langle 0| - |1\rangle_3\langle 1|)(|0\rangle_5\langle 0| - |1\rangle_5\langle 1|)$. If Victor gets $|\xi_1\rangle_{12}$ or $|\xi_2\rangle_{12}$, in both cases, Alice knows in her qubits 3 and 5 she has got a orthogonal-complement copy $|\xi_3\rangle_{35}$ and $|\xi_4\rangle_{35}$ of the original arbitrary unknown state $|u\rangle_{12}$, respectively.

Case III: $\alpha, \beta, \gamma, \delta$ are real and $\eta = 1$

While Victor measures $|\xi_1\rangle_{12}$ or $|\xi_4\rangle_{12}$, the treatment for both cases is same as Case I and Case II, respectively. If Victor's measurement result is $|\xi_2\rangle_{12}$, the state of Alice's qubits 3 and 5 will be left in $\beta|00\rangle_{35} - \alpha|11\rangle_{35} - \delta|01\rangle_{35} + \gamma|10\rangle_{35}$ (except for a overall trivial factor). Then after obtaining two cubits from Victor, Alice performs the unitary operation $U_3 = (|1\rangle_3\langle 0| + |0\rangle_3\langle 1|)(|1\rangle_5\langle 0| - |0\rangle_5\langle 1|)$ on qubits 3, 5 to realize the copy of the original arbitrary unknown state $|u\rangle_{12}$. Based on that proposed above, in Case III, Alice can deterministically realize a perfect clone of the original arbitrary unknown state $|u\rangle$ with assistance of Victor.

Case IV: $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ and $\alpha\gamma = \beta\delta$

In terms of $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$, one can easily obtain $\eta = 1$. So if the measurement outcome is $|\xi_4\rangle_{12}$, Alice can easily clone the state $|u\rangle$ in her qubits 3, 5 via the same process proposed in Case II. Furthermore, in this case, one can also get $(\alpha^*)^{-1} = 4\alpha$, $(\beta^*)^{-1} = 4\beta$, $(\gamma^*)^{-1} = 4\gamma$, $(\delta^*)^{-1} = 4\delta$ and $\alpha^*\beta^* = \gamma^*\delta^*$. Then if the state $|\xi_1\rangle_{12}$ is measured, under Victor's help, Alice knows the state of her qubits 3 and 5, except for a overall trivial factor, will be left in $\beta^*|00\rangle_{35} - \alpha^*|11\rangle_{35} + \delta^*|01\rangle_{35} - \gamma^*|10\rangle_{35}$.It can also be reexpressed as

$$\begin{aligned}
&\beta^*|00\rangle_{35} - \alpha^*|11\rangle_{35} + \delta^*|01\rangle_{35} - \gamma^*|10\rangle_{35}\\
&= \beta^*\delta^*\left(\frac{1}{\delta^*}|00\rangle_{35} - \frac{\alpha^*}{\beta^*\delta^*}|11\rangle_{35} + \frac{1}{\beta^*}|01\rangle_{35} - \frac{\gamma^*}{\beta^*\delta^*}|10\rangle_{35}\right)\\
&= \beta^*\delta^*\left(4\delta|00\rangle_{35} - \frac{1}{4\alpha} \times 16\beta\delta|11\rangle_{35} + 4\beta|01\rangle_{35} - \frac{1}{4\gamma} \times 16\beta\delta|10\rangle_{35}\right)\\
&= 4\beta^*\delta^*(\delta|00\rangle_{35} - \gamma|11\rangle_{35} + \beta|01\rangle_{35} - \alpha|10\rangle_{35}). \qquad (16)
\end{aligned}$$

Then in order to create a perfect copy of the original state in her qubits 3 and 5, Alice only needs to perform the local unitary operation $U_4 = (|1\rangle_3\langle 0| - |0\rangle_3\langle 1|)(|0\rangle_5\langle 0| + |1\rangle_5\langle 1|)$.

In Case IV, if the state $|\xi_2\rangle_{12}$ is measured, then applying the same analysis method, Alice can also get a perfect copy of the original state $|u\rangle$ with Victor's help. The detailed relation between the measurement result, the collapsed states and the appropriate local unitary operation is show in Table 1. So in Case IV, Alice can deterministically realize a perfect clone of the original arbitrary unknown state $|u\rangle$ with assistance of Victor.

Case V: $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ and $\alpha\beta = \delta\gamma$

Case VI, $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ and $\alpha\delta = \beta\gamma$

In both cases, the treatments are very similar to that proposed in Case IV. I will not depict them anymore and summarize detailedly the relation between the measurement result, the collapsed states and the appropriate local unitary operation in Table 1. So in Case V and Case VI, no matter what the measurement result is, Alice can get a perfect clone of the state $|u\rangle$ in her qubits 3, 5 with an appropriate unitary operation corresponding to the two cubits from Victor.

By above analysis, one can see, in general, Alice's perfect clone of the original state can only be realized with a certain probability. Nonetheless, if the state to be clone is chosen from six special ensembles proposed above, the success probability of realizing Alice's perfect clone can be increased to $\frac{1}{2}$ or even 1 after performing an appropriate unitary transformation. Note that, in the process of teleportation, if Alice's measurement outcome is one of the other 15 states (i.e., $|\phi^\pm\rangle_{25}|\phi^\pm\rangle_{13}$, $|\phi^\pm\rangle_{25}|\psi^\pm\rangle_{13}$, $|\psi^\pm\rangle_{25}|\phi^\pm\rangle_{13}$, $|\psi^-\rangle_{25}|\psi^\pm\rangle_{13}$, and $|\psi^+\rangle_{25}|\psi^+\rangle_{13}$), applying the same analysis method as above, Alice can also get either a copy or a complement copy of the original arbitrary unknown state with the help of Victor. While the state to be clone is chosen from six special ensembles proposed above, Alice's perfect clone can also be realized with higher probability or even in a deterministic manner via an appropriate unitary operation.

Now let us further consider the case that non-maximally entangled two-qubit pairs are taken as quantum channel. Suppose that, the qubit pair $(1, 2)$ from Victor is in the state $|p\rangle_{12} = \alpha|00\rangle_{12} + \beta|01\rangle_{12} + \gamma|10\rangle_{12} + \delta|11\rangle_{12}$, where $\alpha, \beta, \gamma, \delta$ are arbitrary complex numbers which satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. In addition, Alice has another two qubit

Table 1 The relation between the measurement result, the collapsed states and the appropriate local unitary operation. GC denotes the case that $\alpha, \beta, \gamma, \delta$ are four arbitrary coefficients; CC denotes which special case the four coefficients belong to; MR denotes Victor's measurement result; CS denotes the collapsed state of Alice's two qubits; U denotes the appropriate local unitary operation

CC	MR	CS	U
GC	$\|\xi_3\rangle_{12}$	$\alpha\|00\rangle_{35}+\beta\|11\rangle_{35}+\gamma\|01\rangle_{35}+\delta\|10\rangle_{35}$	$(\|0\rangle_3\langle 0\|+\|1\rangle_3\langle 1\|)(\|0\rangle_5\langle 0\|+\|1\rangle_5\langle 1\|)$
I	$\|\xi_1\rangle_{12}$	$\beta\|00\rangle_{35}-\alpha\|11\rangle_{35}+\delta\|01\rangle_{35}-\gamma\|10\rangle_{35}$	$(\|1\rangle_3\langle 0\|-\|0\rangle_3\langle 1\|)(\|1\rangle_5\langle 0\|+\|0\rangle_5\langle 1\|)$
II	$\|\xi_4\rangle_{12}$	$\alpha\|00\rangle_{35}+\beta\|11\rangle_{35}-\gamma\|01\rangle_{35}-\delta\|10\rangle_{35}$	$(\|0\rangle_3\langle 0\|-\|1\rangle_3\langle 1\|)(\|0\rangle_5\langle 0\|-\|1\rangle_5\langle 1\|)$
III($\ni$ I, II)	$\|\xi_2\rangle_{12}$	$\beta\|00\rangle_{35}-\alpha\|11\rangle_{35}-\delta\|01\rangle_{35}+\gamma\|10\rangle_{35}$	$(\|1\rangle_3\langle 0\|+\|0\rangle_3\langle 1\|)(\|1\rangle_5\langle 0\|-\|0\rangle_5\langle 1\|)$
IV($\ni$ II)	$\|\xi_1\rangle_{12}$	$\delta\|00\rangle_{35}-\gamma\|11\rangle_{35}+\beta\|01\rangle_{35}-\alpha\|10\rangle_{35}$	$(\|1\rangle_3\langle 0\|-\|0\rangle_3\langle 1\|)(\|0\rangle_5\langle 0\|+\|1\rangle_5\langle 1\|)$
	$\|\xi_2\rangle_{12}$	$\delta\|00\rangle_{35}-\gamma\|11\rangle_{35}-\beta\|01\rangle_{35}+\alpha\|10\rangle_{35}$	$(\|1\rangle_3\langle 0\|+\|0\rangle_3\langle 1\|)(\|0\rangle_5\langle 0\|-\|1\rangle_5\langle 1\|)$
V($\ni$ II)	$\|\xi_1\rangle_{12}$	$\alpha\|00\rangle_{35}-\beta\|11\rangle_{35}+\gamma\|01\rangle_{35}-\delta\|10\rangle_{35}$	$(\|0\rangle_3\langle 0\|-\|1\rangle_3\langle 1\|)(\|0\rangle_5\langle 0\|+\|1\rangle_5\langle 1\|)$
	$\|\xi_2\rangle_{12}$	$\alpha\|00\rangle_{35}-\beta\|11\rangle_{35}-\gamma\|01\rangle_{35}+\delta\|10\rangle_{35}$	$(\|0\rangle_3\langle 0\|+\|1\rangle_3\langle 1\|)(\|0\rangle_5\langle 0\|-\|1\rangle_5\langle 1\|)$
VI($\ni$ II)	$\|\xi_1\rangle_{12}$	$\gamma\|00\rangle_{35}-\delta\|11\rangle_{35}+\alpha\|01\rangle_{35}-\beta\|10\rangle_{35}$	$(\|0\rangle_3\langle 0\|-\|1\rangle_3\langle 1\|)(\|0\rangle_5\langle 1\|+\|1\rangle_5\langle 0\|)$
	$\|\xi_2\rangle_{12}$	$\gamma\|00\rangle_{35}-\delta\|11\rangle_{35}-\alpha\|01\rangle_{35}+\beta\|10\rangle_{35}$	$(\|0\rangle_3\langle 0\|+\|1\rangle_3\langle 1\|)(\|0\rangle_5\langle 1\|-\|1\rangle_5\langle 0\|)$

pairs $(3,4)$ and $(5,6)$. Without loss of generality, suppose the two pairs $(3,4)$ and $(5,6)$ are in other same Bell states:

$$|t\rangle_{34} = a|00\rangle_{34} + b|11\rangle_{34} \qquad (|a|^2+|b|^2=1), \tag{17}$$

$$|f\rangle_{56} = c|00\rangle_{34} + d|11\rangle_{34} \qquad (|c|^2+|d|^2=1), \tag{18}$$

where a, b, c and d are nonzero real numbers, and $|a| \geq |b|$ and $|c| \geq |d|$ are assumed. Hence the joint state of the six qubits in Alice's site is

$$\begin{aligned} |w'\rangle_{123456} &= |p\rangle_{12}|t\rangle_{34}|f\rangle_{56} \\ &= (\alpha|00\rangle_{12} + \beta|01\rangle_{12} + \gamma|10\rangle_{12} + \delta|11\rangle_{12})(a|00\rangle_{34} \\ &\quad + b|11\rangle_{34})(c|00\rangle_{56} + d|11\rangle_{56}). \end{aligned} \tag{19}$$

It can be rewritten as

$$\begin{aligned} |w'\rangle_{123456} &= \frac{1}{2}|\phi^+\rangle_{i1}|\phi^+\rangle_{j3}(\alpha ac|00\rangle_{24} + \beta ad|01\rangle_{24} + \gamma bc|10\rangle_{24} + \delta bd|11\rangle_{24}) \\ &+ \frac{1}{2}|\phi^-\rangle_{i1}|\phi^+\rangle_{j3}(\alpha ac|00\rangle_{24} + \beta ad|01\rangle_{24} - \gamma bc|10\rangle_{24} - \delta bd|11\rangle_{24}) \\ &+ \frac{1}{2}|\phi^+\rangle_{i1}|\phi^-\rangle_{j3}(\alpha ac|00\rangle_{24} - \beta ad|01\rangle_{24} + \gamma bc|10\rangle_{24} - \delta bd|11\rangle_{24}) \\ &+ \frac{1}{2}|\phi^-\rangle_{i1}|\phi^-\rangle_{j3}(\alpha ac|00\rangle_{24} - \beta ad|01\rangle_{24} - \gamma bc|10\rangle_{24} + \delta bd|11\rangle_{24}) \\ &+ \frac{1}{2}|\psi^+\rangle_{i1}|\phi^+\rangle_{j3}(\alpha bc|10\rangle_{24} + \beta bd|11\rangle_{24} + \gamma ac|00\rangle_{24} + \delta ad|01\rangle_{24}) \\ &+ \frac{1}{2}|\psi^-\rangle_{i1}|\phi^+\rangle_{j3}(\alpha bc|10\rangle_{24} + \beta bd|11\rangle_{24} - \gamma ac|00\rangle_{24} - \delta ad|01\rangle_{24}) \\ &+ \frac{1}{2}|\psi^+\rangle_{i1}|\phi^-\rangle_{j3}(\alpha bc|10\rangle_{24} - \beta bd|11\rangle_{24} + \gamma ac|00\rangle_{24} - \delta ad|01\rangle_{24}) \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}|\psi^-\rangle_{i1}|\phi^-\rangle_{j3}(\alpha bc|10\rangle_{24}-\beta bd|11\rangle_{24}-\gamma ac|00\rangle_{24}+\delta ad|01\rangle_{24})\\
&+\frac{1}{2}|\phi^+\rangle_{i1}|\psi^+\rangle_{j3}(\alpha ad|01\rangle_{24}+\beta ac|00\rangle_{24}+\gamma bd|11\rangle_{24}+\delta bc|10\rangle_{24})\\
&+\frac{1}{2}|\phi^-\rangle_{i1}|\psi^+\rangle_{j3}(\alpha ad|01\rangle_{24}+\beta ac|00\rangle_{24}-\gamma bd|11\rangle_{24}-\delta bc|10\rangle_{24})\\
&+\frac{1}{2}|\phi^+\rangle_{i1}|\psi^-\rangle_{j3}(\alpha ad|01\rangle_{24}-\beta ac|00\rangle_{24}+\gamma bd|11\rangle_{24}-\delta bc|10\rangle_{24})\\
&+\frac{1}{2}|\phi^-\rangle_{i1}|\psi^-\rangle_{j3}(\alpha ad|01\rangle_{24}-\beta ac|00\rangle_{24}-\gamma bd|11\rangle_{24}+\delta bc|10\rangle_{24})\\
&+\frac{1}{2}|\psi^+\rangle_{i1}|\psi^+\rangle_{j3}(\alpha bd|11\rangle_{24}+\beta bc|10\rangle_{24}+\gamma ad|01\rangle_{24}+\delta ac|00\rangle_{24})\\
&+\frac{1}{2}|\psi^-\rangle_{i1}|\psi^+\rangle_{j3}(\alpha bd|11\rangle_{24}+\beta bc|10\rangle_{24}-\gamma ad|01\rangle_{24}-\delta ac|00\rangle_{24})\\
&+\frac{1}{2}|\psi^+\rangle_{i1}|\psi^-\rangle_{j3}(\alpha bd|11\rangle_{24}-\beta bc|10\rangle_{24}+\gamma ad|01\rangle_{24}-\delta ac|00\rangle_{24})\\
&+\frac{1}{2}|\psi^-\rangle_{i1}|\psi^-\rangle_{j3}(\alpha bd|11\rangle_{24}-\beta bc|10\rangle_{24}-\gamma ad|01\rangle_{24}+\delta ac|00\rangle_{24}).
\end{aligned}
\tag{20}
$$

Similarly, in order to clone and orthogonally complement the arbitrary unknown two-qubit entangled state $|p\rangle$, Alice should first accomplish a quantum teleportation process of an *arbitrary* (*not a special class of*) unknown two-qubit entangled state. In the teleportation process, POVM will be used instead of the more commonly used collective unitary transformation. That is, Alice performs projective Bell-state measurements on the qubit pairs (2, 5) and (1, 3) respectively, and if Alice measures $|\phi^+\rangle_{13}|\phi^+\rangle_{25}$, then the exact state of qubits 4 and 6, according to (20), is

$$|T\rangle_{46}=\frac{1}{2}(\alpha ac|00\rangle_{46}+\beta ad|01\rangle_{46}+\gamma bc|10\rangle_{46}+\delta bd|11\rangle_{46}). \tag{21}$$

In order to accomplish the teleportation, Alice firstly introduces two auxiliary qubits m and n which are in the state $|00\rangle_{mn}$. As a consequence, the state of the qubits 4, 6, m and n now in Bob's site is

$$|T\rangle_{46}|00\rangle_{mn}=\frac{1}{2}(\alpha ac|00\rangle_{46}+\beta ad|01\rangle_{46}+\gamma bc|10\rangle_{46}+\delta bd|11\rangle_{46})|00\rangle_{mn}. \tag{22}$$

After the incorporation of the two auxiliary qubits, Alice then performs two controlled-not (CNOT) operations with qubits 4 and 6 as the controlled qubits while the auxiliary qubits m and n as the target qubits, respectively. The two CNOT operations transform the state in (22) into the following form

$$|T'\rangle_{46mn}=\frac{1}{2}(\alpha ac|0000\rangle_{46mn}+\beta ad|0101\rangle_{46mn}+\gamma bc|1010\rangle_{46mn}+\delta bd|1111\rangle_{46mn}). \tag{23}$$

This state can be further rewritten as

$$|T'\rangle_{46mn}=\frac{1}{8}(|K_1\rangle_{46}|Q_1\rangle_{mn}+|K_2\rangle_{46}|Q_2\rangle_{mn}+|K_3\rangle_{46}|Q_3\rangle_{mn}+|K_4\rangle_{46}|Q_4\rangle_{mn}), \tag{24}$$

where

$$|K_1\rangle_{46} = \alpha|00\rangle_{46} + \beta|01\rangle_{46} + \gamma|10\rangle_{46} + \delta|11\rangle_{46}, \tag{25}$$

$$|Q_1\rangle_{mn} = ac|00\rangle_{mn} + ad|01\rangle_{mn} + bc|10\rangle_{mn} + bd|11\rangle_{mn}, \tag{26}$$

$$|K_2\rangle_{46} = (\alpha|00\rangle_{46} + \beta|01\rangle_{46} - \gamma|10\rangle_{46} - \delta|11\rangle_{46}, \tag{27}$$

$$|Q_2\rangle_{mn} = ac|00\rangle_{mn} + ad|01\rangle_{mn} - bc|10\rangle_{mn} - bd|11\rangle_{mn}, \tag{28}$$

$$|K_3\rangle_{46} = \alpha|00\rangle_{46} - \beta|01\rangle_{46} + \gamma|10\rangle_{46} - \delta|11\rangle_{46}, \tag{29}$$

$$|Q_3\rangle_{mn} = ac|00\rangle_{mn} - ad|01\rangle_{mn} + bc|10\rangle_{mn} - bd|11\rangle_{mn}, \tag{30}$$

$$|K_4\rangle_{46} = \alpha|00\rangle_{46} - \beta|01\rangle_{46} - \gamma|10\rangle_{46} + \delta|11\rangle_{46}, \tag{31}$$

$$|Q_4\rangle = ac|00\rangle_{mn} - ad|01\rangle_{mn} - bc|10\rangle_{mn} + bd|11\rangle_{mn}. \tag{32}$$

From the (24), one can see that, Alice will get the state $|K_i\rangle_{46}$ $(i = 1, 2, 3, 4)$ of his qubits 4 and 6 provided that $|Q_i\rangle_{mn}$ $(i = 1, 2, 3, 4)$ are obtained via measurements on his auxiliary qubits m and n. Unfortunately, it is impossible to precisely and deterministically distinguish the four $|Q\rangle_{mn}$ states. Nevertheless, the discrimination can be achieved in a probabilistic manner by making an optimal POVM measurement [36, 38] on the ancillary qubits m and n as follows,

$$P_1 = \frac{1}{x}|M_1\rangle\langle M_1|, \tag{33}$$

$$P_2 = \frac{1}{x}|M_2\rangle\langle M_2|, \tag{34}$$

$$P_3 = \frac{1}{x}|M_3\rangle\langle M_3|, \tag{35}$$

$$P_4 = \frac{1}{x}|M_4\rangle\langle M_4|, \tag{36}$$

$$P_5 = I - \frac{1}{x}\sum_{i=1}^{4}|M_i\rangle\langle M_i|. \tag{37}$$

Here

$$|M_1\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{ac}|00\rangle + \frac{1}{ad}|01\rangle + \frac{1}{bc}|10\rangle + \frac{1}{bd}|11\rangle\right)_{mn}, \tag{38}$$

$$|M_2\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{ac}|00\rangle + \frac{1}{ad}|01\rangle - \frac{1}{bc}|10\rangle - \frac{1}{bd}|11\rangle\right)_{mn}, \tag{39}$$

$$|M_3\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{ac}|00\rangle - \frac{1}{ad}|01\rangle + \frac{1}{bc}|10\rangle - \frac{1}{bd}|11\rangle\right)_{mn}, \tag{40}$$

$$|M_4\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{ac}|00\rangle - \frac{1}{ad}|01\rangle - \frac{1}{bc}|10\rangle + \frac{1}{bd}|11\rangle\right)_{mn}, \tag{41}$$

$$\xi = \frac{1}{(ac)^2} + \frac{1}{(ad)^2} + \frac{1}{(bc)^2} + \frac{1}{(bd)^2} = \frac{1}{(1-b^2)b^2(1-d^2)d^2}, \tag{42}$$

I is an identity operator, x is a coefficient relating to a, b, c and d, which satisfies $1 \leq x \leq 4$ and should be able to assure P_5 to be a positive operator. To exactly determine x, we would like to rewrite the five operators P_1, P_2, P_3, P_4 and P_5 in the matrix form

$$P_1 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ac)^2} & \frac{1}{acad} & \frac{1}{acbc} & \frac{1}{acbd} \\ \frac{1}{acad} & \frac{1}{(ad)^2} & \frac{1}{adbc} & \frac{1}{adbd} \\ \frac{1}{acbc} & \frac{1}{adbc} & \frac{1}{(bc)^2} & \frac{1}{bcbd} \\ \frac{1}{acbd} & \frac{1}{adbd} & \frac{1}{bdbc} & \frac{1}{(bd)^2} \end{pmatrix}, \tag{43}$$

$$P_2 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ac)^2} & \frac{1}{acad} & -\frac{1}{acbc} & -\frac{1}{acbd} \\ \frac{1}{acad} & \frac{1}{(ad)^2} & -\frac{1}{adbc} & -\frac{1}{adbd} \\ -\frac{1}{acbc} & -\frac{1}{adbc} & \frac{1}{(bc)^2} & \frac{1}{bcbd} \\ -\frac{1}{acbd} & -\frac{1}{adbd} & \frac{1}{bdbc} & \frac{1}{(bd)^2} \end{pmatrix}, \tag{44}$$

$$P_3 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ac)^2} & -\frac{1}{acad} & \frac{1}{acbc} & -\frac{1}{acbd} \\ -\frac{1}{acad} & \frac{1}{(ad)^2} & -\frac{1}{adbc} & \frac{1}{adbd} \\ \frac{1}{acbc} & -\frac{1}{adbc} & \frac{1}{(bc)^2} & -\frac{1}{bcbd} \\ -\frac{1}{acbd} & \frac{1}{adbd} & -\frac{1}{bdbc} & \frac{1}{(bd)^2} \end{pmatrix}, \tag{45}$$

$$P_4 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ac)^2} & -\frac{1}{acad} & -\frac{1}{acbc} & \frac{1}{acbd} \\ -\frac{1}{acad} & \frac{1}{(ad)^2} & \frac{1}{adbc} & -\frac{1}{adbd} \\ \frac{1}{acbc} & -\frac{1}{adbc} & -\frac{1}{(bc)^2} & \frac{1}{bcbd} \\ -\frac{1}{acbd} & \frac{1}{adbd} & \frac{1}{bdbc} & -\frac{1}{(bd)^2} \end{pmatrix}, \tag{46}$$

$$P_5 = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}, \tag{47}$$

where

$$A = 1 - \frac{4}{x\xi(ac)^2}, \tag{48}$$

$$B = 1 - \frac{4}{x\xi(ad)^2}, \tag{49}$$

$$C = 1 - \frac{4}{x\xi(bc)^2}, \tag{50}$$

$$D = 1 - \frac{4}{x\xi(bd)^2}. \tag{51}$$

Obviously, the coefficient x should be chosen such that all the diagonal elements of P_5 are nonnegative. Alice performs the above POVM operation on her auxiliary qubits m and n. She can obtain P_i $(i = 1, 2, 3, 4)$ with probability p, where

$$p = p(P_i) = {}_{46mn}\langle T'|P_i|T'\rangle_{46mn} = {}_{mn}\langle Q_i|P_i|Q_i\rangle_{mn}/64 = \frac{1}{4x\xi} \quad (i = 1, 2, 3, 4). \tag{52}$$

For each P_i $(i = 1, 2, 3, 4)$ Alice can positively conclude that the corresponding state of qubits m and n is $|Q_i\rangle_{mn}$ $(i = 1, 2, 3, 4)$. However, if Alice's POVM outcome is P_5 (such probability is $1-\frac{1}{x\xi}$), then she can not infer which state the qubits m and n is in. Once Alice determines the $|Q_i\rangle_{mn}$ $(i=1,2,3,4)$, this means she also knows the state of her qubits 4 and 6 is $|K_i\rangle_{46}$ $(i=1,2,3,4)$. In this case, she can perform an appropriate unitary operation to reconstruct the original unknown state in her qubits 4 and 6 now. That is, she needs to perform the unitary operation I $(\sigma_z^4 I_6, I_4\sigma_z^6, \sigma_z^4\sigma_z^6)$ if the state in her qubits 4 and 6 is $|K_1\rangle_{46}(|K_2\rangle_{46}, |K_3\rangle_{46}, |K_4\rangle_{46})$.

Above we only consider the case that Alice's Bell-state measurement outcome is $|\phi^+\rangle_{13}|\phi^+\rangle_{25}$. As matter of fact, her outcome can be one of the other 15 Bell-state pairs (i.e., $|\psi^\pm\rangle_{i1}|\phi^\pm\rangle_{j3}$,$|\psi^\pm\rangle_{i1}|\psi^\pm\rangle_{j3}$, $|\phi^\pm\rangle_{i1}|\psi^\pm\rangle_{j3}$,$|\phi^\pm\rangle_{i1}|\phi^-\rangle_{j3}$ and $|\phi^-\rangle_{i1}|\phi^+\rangle_{j3}$). For each Bell-state pair, applying the same method demonstrated before, Alice can also probabilistically achieve her goal of teleporting the state $|p\rangle$ to her qubits 4 and 6. In all, the success probability of teleportation is

$$P=\frac{16}{x\xi}=\frac{16(1-b^2)(1-d^2)}{x}b^2d^2. \tag{53}$$

From (53), one can see that the teleportation probability depends on the parameter x. As mentioned before, x can be varied in from 1 to 4, however, it should still be carefully chosen such that P_5 (see (47)) is a nonnegative operator. If the quantum channel is made of two Bell states, i.e., $a=b=c=d=\frac{1}{\sqrt{2}}$, then one can choose $x=1$ such that P_5 is a zero operator. In this case, the total probability is 1, and the present probabilistic teleportation scheme becomes a deterministic one which is proposed above. After accomplishing the quantum teleportation process, For Alice's each measurement result, applying the same analysis method as above, Alice can also probabilistically get either a copy or a complement copy of the original arbitrary unknown state with the help of Victor. While the state to be clone is chosen from six special ensembles proposed above, Alice's perfect clone can be realized with higher probability or even in a deterministic manner with an appropriate unitary operation.

To summarize, by using the POVM instead of the more commonly used collective unitary operation in the quantum teleportation process, I have successfully generalized the protocols proposed by Zhan recently [35] to clone or anti-clone an *arbitrary* unknown two-qubit entangled state. With the assistance of the unknown-state preparer Victor, the present protocols can produce perfect copy or orthogonal-complement copies of an *arbitrary* unknown two-qubit entangled state via classical and quantum channels. Furthermore, I also show that, while the state to be cloned is chosen from six special ensembles, Alice's goal of realizing a perfect clone can be achieved with higher probability or even in a deterministic manner after performing an appropriate unitary operation.

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